


DOI [https://doi.org/10.58442/2218-7650-2023-23\(52\)-65-81](https://doi.org/10.58442/2218-7650-2023-23(52)-65-81)  
UDC 372.851

**Esmira Jafarli,**

Doctor of Philosophy in Pedagogy,  
Assistant professor of Ganja State University.  
Baku, Azerbaijan.

 <https://orcid.org/0000-0003-3619-4356>  
[gulizade.gunel@mail.ru](mailto:gulizade.gunel@mail.ru)

**Jabarud Mammadova,**

Head teacher of Ganja State University.  
Baku, Azerbaijan.

**Parvin Mustafayeva,**

Teacher of Ganja State University.  
Baku, Azerbaijan.

## **OPTIMAL WAYS OF TEACHING THE TOPIC «TRIGONOMETRIC EQUATIONS» IN THE SCHOOL MATHEMATICS COURSE**

**Abstract.** The article selected for research is dedicated to the optimal methods of teaching trigonometric equations in the high school mathematics course based on the curriculum. In the article, it is also confirmed that the studied material is the logical conclusion of the trigonometry course, the final result of the passed theoretical knowledge. The topic of trigonometric equations and inequalities provides excellent examples and problems of integration between algebra, geometry and other disciplines. Trigonometry is closely related to both geometry and algebra in its characteristics, and this connection is reflected in the content of the school mathematics course. Thus, trigonometry is used in geometry in metric relationships in right triangles, in the definition of the sine, cosine, tangent and cotangent of an acute angle, in the theorem of sines and cosines, in the relationship formulas between sides and radii when circles are drawn inside and outside regular polygons, in area formulas of many geometric figures, etc., in algebra and in the course, the main formulas of trigonometry, trigonometric equations and inequalities, etc. are given in the sections. Our main goal in the dissertation is to explain the place and importance of trigonometric equations and inequalities in the school mathematics course, to show the role of trigonometric equations in the equation family, and to determine the methodical features of its solution. At the same time, in the process of teaching trigonometric

equations and inequalities, the solution of algebraic equations included in the family of equations is also considered, and this requires both the teacher and the student to teach and learn the high school mathematics course in a complete, integral way.

**Keywords:** mathematics course; trigonometric equations; teaching process; students; teacher; algebra; geometry; school.

## **INTRODUCTION / ВСТУП**

**Formulation of the problem.** Optimal ways of teaching the topic «Trigonometric Equations» in the school mathematics course – the topic is one of the complex, extensive areas studied in the secondary school of mathematics in grades V–XI. Before passing the trigonometric equations, students mastered the lessons and materials of the lower grades, which prepare the basic foundations of this subject. The radian measure of an angle, calculating the length of an arc, metric ratios in a right triangle, monotonicity of functions, etc., such topics create a positive basis for understanding and understanding the methods for solving trigonometric equations. The article also shows more efficient and optimal methods for solving each equation and gives various recommendations. Integration, which is one of the basic principles of the educational program of the curriculum, and its application has been given enough space in the topic, and the application of trigonometry in various fields has been shown as an interesting issue.

For many years, various subjects, including mathematics, have been taught in general education schools on the basis of the curriculum. The main goal of the new curriculum for the secondary school mathematics course is to prepare students who are educated, versatile, with a broad worldview, with high mental abilities, logical thinking, determination, judgment and the ability to draw conclusions. In the subject curricula in mathematics, each subject received a more complete content, taking into account the positive indicators of previous years, and enough space was allocated for classes with practical content, which must be solved logically.

**Analysis of major research and publications.** The role and significance of problem solving in the school course of mathematics has always been the subject of research by leading teachers, mathematicians-methodologists. Since the twentieth century, M. Abdulkarimov [1], M. Ashurov [2], B. Aliev [3], S. Hamidov [5] and others attached great importance to problem solving in the development of secondary mathematical education and always kept it in the center of attention in the training of young mathematicians. At the end of the

20th and the beginning of the 21st century, the school of methods of teaching mathematics became widespread in Azerbaijan. S. Gamidov [5], I. Rustamov [17], S. Huseynova [9], M. Mardanov [15] and others in their classes brought to the fore the problem of the connection between learning and life.

### AIM AND TASKS / МЕТА ТА ЗАВДАННЯ

The **purpose** of the study is to develop a methodological system based on a system of tasks taken from life to ensure the mathematical preparation of students.

In this regard, the **main objectives** of the study are as follows:

- reveal the role and significance of the problem in mathematical education;
- to reveal didactic functions of questions;
- to determine the possibilities of mathematics textbooks for grades V-VI;
- discover the use of mental operations in solving problems;
- to determine the ways of choosing questions, the content of which is taken from life;
- to determine the stages of solving the problem, methods of choosing modeling tools;
- identify the difficulties that students face in solving problems and ways to overcome them;
- in accordance with the purpose of the problem under study, taking into account the psychological, pedagogical and scientific requirements, to prepare a system of tasks and a methodology for teaching their solution.

### THE THEORETICAL BACKGROUNDS / ТЕОРЕТИЧНІ ОСНОВИ ДОСЛІДЖЕННЯ

It should be noted that before moving on to trigonometric equations, the «concept of trigonometry» begins with the concepts of  $\sin\alpha$ ,  $\cos\alpha$ ,  $\operatorname{tg}\alpha$  and  $\operatorname{ctg}\alpha$ , which appear in grades 8–9 of high school as relationships between the sides and angles of a right triangle. Later in the same lessons, students learn how to transform these functions into one another in a right triangle and their meanings at certain angles.

A comprehensive and in-depth study of the trigonometry course in grade 10 «Angle of rotation, arc length, trigonometric functions, formulas for their transformation, summation theorems, results from them», etc. begins with the teaching of subjects. The properties of trigonometric functions, their graphs, determining the ranges of increment and decrease, calculating values on the coordinate plane and on the unit circle, the ability to describe functions are

studied before the topic «trigonometric equations», the relevant knowledge and skills have already been formed among students.

Effective and efficient teaching of trigonometric equations primarily depends on the correct organization of teaching the simplest trigonometric equations. At this stage of training, the teacher should try to reasonably understand both the theory of equations, the properties of trigonometric functions, especially those properties that will be used in solving trigonometric equations, and the features of solving trigonometric equations that differ from other types of equations, the principle of proportionality of trigonometric equations, the number of units trigonometric equations. The classification of simple trigonometric equations must be correctly given, the rule for writing the found cubes as a whole must be correctly conveyed to students. This stage of learning is the discovery of the main content of the cube theorem, the connection of the theorem with the monotonicity of the function and the existence of this property of trigonometric functions, the explanation of simple trigonometric equations in the form of a cube, arcsine, arccosine, arctanensis, arccotangent and their many-valued nature. It is also very important to give a geometric explanation for the fact that simple trigonometric equations have no solution.

In all stable textbooks and tutorials, the simplest trigonometric equations are treated as equations, and the teaching of these topics in textbooks begins with solving the equation. The main methodological features of teaching the subject are that, unlike previously taught equations, the solution of this type of equations is associated with great difficulties. Thus, the use of self-help methods by students is reduced, students for the first time get acquainted with the concept of serial solutions, see a real-inductive explanation for finding cubes, face the problem of choosing the desired cubes from the set, serial cubes were found, therefore, the subject of trigonometric equations from the first lessons, the teacher requires high professionalism and methodological skills. In the first lesson, the teacher should very meaningfully explain to students the square theorem, the importance of the condition for increasing or decreasing a function on an interval, decreasing an arbitrary value of the function  $f(x)$  in the interval under consideration, and the fact that the expression  $f(x) \in a$  is quadratic to the interval from the point requirements view, should be able to do so. After this explanation, new concepts are introduced, such as arcsine, arccosine, etc., which are of great importance in solving trigonometric equations they must be very careful in their understanding, they must be based on the knowledge they have acquired before.

One of the biggest difficulties in solving trigonometric equations is the question of how to solve a given trigonometric equation. In order to overcome

this difficulty, the teacher should first of all classify equations, convey this classification to students in a mnemonic way, require them to correctly know the names of the equations included in the classification, and it should be explained that each type of trigonometric equation has its own solution method. At this stage of learning, you should consider the problem of solving the same trigonometric equation in several ways.

In our opinion, at this stage, the teacher should prepare cards for each student and write on these cards trigonometric equations covering all types, and ask students to determine their types, since experiments show that if the student does not know what type of trigonometric equation the solution consists of in that the student must be able to solve the equation, faces a dilemma, but when the student knows what type of trigonometric equation this trigonometric equation belongs to. Therefore, it is necessary to pay special attention to the definition of types of trigonometric equations. Completing the solution of trigonometric equations is also very important. The checks carried out show that often the solution of trigonometric equations is not completed, that is, the phase of connecting the roots remains incomplete. Ultimately, this means that the equation remains unsolved.

The results of experiments and studies show that students face great difficulties in solving trigonometric equations and find it difficult to choose ways to simplify the given trigonometric equations using trigonometric transformations. One of the main reasons for this is due to the poor knowledge of trigonometric formulas in the textbook, and the other reason and the most important reason is that trigonometric equations are not classified and researched before solving trigonometric equations. In our opinion, both trigonometric formulas should be very clearly grouped and repeatedly and meaningfully explained to these students. In our opinion, it is very important to divide the trigonometric formulas into 9 large groups based on the trigonometric material included in the school curriculum, and determine the importance and scope of each group. It also simplifies the classification and solution of trigonometric equations.

In our opinion, it is methodically incorrect to carry out the educational process by solving mixed-type trigonometric equations. Therefore, we suggest that in the first lesson, i.e., in the next lesson after the ability to solve simple trigonometric equations, the teacher should give the meaning of the classification of trigonometric equations in the form of a lecturer, the basis for this classification and, finally, the classification table, then move on to solving specific examples and each time to investigate what type of trigonometric equation needs to be solved. In our opinion,

the following classification of trigonometric equations has a positive effect on improving the quality of education.

- 1) Trigonometric equations are introduced into the algebraic equation.
- 2) Homogeneous trigonometric equations.
- 3) Trigonometric equations solved by the factorization method.
- 4) Solution of trigonometric equations using the condition of equality of the trigonometric equations of the same name.
- 5) The solution of trigonometric equations using the formulas for the transformation of the sum and difference.
- 6) Solving trigonometric equations using addition formulas and summation formulas.
- 7) Trigonometric equations are solved using reduction formulas.
- 8) Trigonometric equations of the form  $a\sin x + b\cos x = c$ .

After this classification, the method of solving each type of trigonometric equation and the differential choice of the corresponding type of research system is suitable for the purpose.

Trigonometric equations are introduced into the algebraic equation.

Students should understand that a large class of trigonometric equations are solved by reducing them to an algebraic equation, and at the same time, students should know what trigonometric equations look like in algebraic form, equations such as the following:

$$a\sin 2x + b\sin x + c = 0$$

$$a\cos 3x + b\cos x + c = 0$$

$a\operatorname{tg} 43x + b\operatorname{tg} 23x + c = 0$  and so on, including equations such as Indeed, in equations of this type

$$\sin x = t$$

$$\cos x = y$$

The replacement  $\operatorname{tg} 3x = z$  gives the following algebraic equations.

$$at^2 + bt + c = 0$$

$$ay^3 + by + c = 0$$

$$at^4 + bt^2 + c = 0$$

By solving each of these equations, we find  $\sin x = t$ ,  $\cos x = y$ , and  $\operatorname{tg} 3x$ , and then we find the variable by solving these equations. It should be explained to students that there are trigonometric equations that at first glance do not look like algebraic equations, but after a simple trigonometric transformation they become algebraic equations.

$$a\sin 2x + b\cos x + c = 0$$

$$atgx + bctgx = 0$$

The form  $a\cos 2x + b\sin x + c = 0$  is not like an algebraic equation because each equation involves two functions instead of one, but these equations are converted to an algebraic equation by a simple transformation. Really

$$\sin x = 1 - \cos 2x,$$

$$\cos 2x = 1 - \sin 2x,$$

Using the identities  $ctgx = 1/tgx$ , the required equations are obtained. In our opinion, the teacher should write some trigonometric equations on the board and ask the students which one is the equation reduced to an algebraic equation and ask them to justify their opinion. After that, he should move on to solving specific examples and solve very simple examples first.

Example 1. Solve the equation  $\cos 2x + 3\sin x = 2$ . This example at first glance does not look like an algebraic equation, why? Because there are two functions and the arguments are different.

However, after simple transformations, this equation easily turns into an algebraic equation. we know it.

$$\cos 2x = 1 - 2\sin^2 x \text{ Then}$$

$$1 - 2\sin^2 x + 3\sin x = 2$$

$$2\sin^2 x - 3\sin x + 1 = 0 \text{ where } \sin x = t \text{ after substitution}$$

A quadratic equation  $2t^2 - 3t + 1 = 0$  is obtained, which is easily solved

We get  $t_1 = 1/2$  and  $t_2 = 1$ , and then

$$\sin x = 1/2 \quad x = (-1)^n \pi/4, \quad n \in \mathbb{Z}$$

$$\text{we get } \sin x = 1 \quad x = \pi/2 + 2\pi n, \quad n \in \mathbb{Z}.$$

In our opinion, in order to solve these types of equations, students should master the following formulas.  $\sin^2 x + \cos^2 x = 1$

$$1) \quad tgx = \frac{\sin x}{\cos x}$$

$$2) \quad ctgx = \frac{\cos x}{\sin x}$$

$$3) \quad ctgx = \frac{1}{tgx}$$

$$4) \quad 1 + tg^2 x = \frac{1}{\cos^2 x}$$

$$5) \quad 1 + ctg^2 x = \frac{1}{\sin^2 x}$$

$$6) \quad 1 + \cos 2x = 2\cos^2 x$$

$$7) \quad 1 - \cos 2x = 2\sin^2 x$$

$$8) \quad tg 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$9) \sin 2x = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}$$

$$10) \cos 2x = 1 - \operatorname{tg}^2 x$$

$$11) \sin 2x = 2 \sin x \cos x$$

$$12) \cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

In addition, they must know the transformation formulas and formulas for the roots of the simplest trigonometric equations.

To master the solution of these types of equations, the teacher must prepare a learning system.

Homogeneous trigonometric equations include the following types of equations.

$$a \sin x + b \cos x = 0$$

$$a \sin 2x + b \sin x \cos x + c \cos 2x = 0$$

$$a \sin 3x + b \sin 2x \cos x + c \cos 2x \sin x + d \cos 3x = 0$$

These equations are homogeneous with respect to  $\sin x$  and  $\cos x$ , respectively. In other words, such equations are called a homogeneous trigonometric equation, the sum of the degrees of the functions included in the equation is the same in all of them. This sum is called the degree of a homogeneous trigonometric equation. The powers of the homogeneous trigonometric equations under consideration are equal to one, two, and three, respectively. By dividing these types of equations by  $\cos^k x$  (the power of the  $k$ -equation), the homogeneous trigonometric equation is converted into an algebraic equation. For example.

Let's look at the equation  $a \sin 2x + b \sin x \cos x + c \cos 2x = 0$ . Divide the equation by  $\cos^2 x$ :

$$a \operatorname{tg} 2x + b \operatorname{tg} x + c = 0$$

Here it is very important to explain the situation to the students. Since  $\cos x \neq 0$ , we can divide by  $\cos^2 x$ . If  $\cos x = 0$ , then  $\sin x = 0$ , as the equations show. It's impossible. If this were the case, then the identity  $\sin^2 x + \cos^2 x = 1$  would lose its meaning. In our opinion, this case should be specially explained. That is, students should fully understand that  $\sin x$  and  $\cos x$  cannot be zero for the same argument value.

By substituting  $\operatorname{tg} x = y$  from the equation  $a \operatorname{tg} 2x + b \operatorname{tg} x + c = 0$ , we get the equation  $ay^2 + by + c = 0$ , which is easily solved. When explaining the solution of these types of equations, it is necessary to clarify that at first glance the equation may not look homogeneous, but it can still be reduced to a homogeneous trigonometric equation using simple trigonometric transformations. For



example:  $a\sin 2x + b\sin x \cos x + c\cos 2x = d$  is not a homogeneous equation, but it can be easily reduced to a homogeneous equation. Indeed:  $d = d \cdot 1 = d(\cos^2 x + \sin^2 x)$

Hence the following equation:

$a\sin 2x + b\sin x \cos x + c\cos 2x = d(\cos^2 x + \sin^2 x)$  with simple transformations and dividing each side by  $\cos 2x$ .

The algebraic equation  $(a-d)\tan 2x + b\tan x + c = 0$  is obtained and the students already know its solution. After that, it is necessary to move on to solving examples, that is, to master the solution of this type of equations, the teacher must prepare a system of classes.

Trigonometric equations solved by the factorization method.

The solution of these types of equations requires students to be well aware of all methods of factoring algebraic expressions, that is, each student must be well aware of the methods of taking the common factor out of brackets, groupings, abbreviated multiplication formulas and artificial methods for obtaining their results, and in addition, he must know the formulas that we wrote at the beginning of the paragraph and additionally the following formulas/

$$1) \operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \operatorname{tg}\beta}$$

$$2) \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$3) \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

After a detailed explanation by the teacher, several trigonometric equations are written on the blackboard, including the types of equations that the students already know, and the students must explain what type these equations are or say that they do not know. After this work will help to solve examples. It is better to choose a learning system that uses methods for dividing possible algebraic expressions into points, it is known that this requires both professionalism and skill from teachers.

Trigonometric equations are solved using trigonometric equations of the same name with the condition of equality of functions. It is necessary to explain to students that the solution of many trigonometric equations is reduced to the equality of trigonometric functions of the same name. These types of equations are solved based on the condition of equality of trigonometric functions of the same name. That is, based on such conditions that both angles  $\alpha$  and  $\beta$  satisfy this condition.

The fulfillment of these conditions, i.e.  $\sin\alpha = \sin\beta$ ;  $\cos\alpha = \cos\beta$ ;  $\operatorname{tg}\alpha = \operatorname{tg}\beta$ , is proved in the form of a theorem, and there are three theorems in total. Let us prove one of these theorems.

**Theorem 1.** A necessary and sufficient condition for the equality of the sines of two angles is one of the following conditions: the difference of these angles must be equal to  $\pi$  times an even number, or the sum of the angles must be equal to  $\pi$  times an even number. odd number. Proof of the necessary condition:

Given:  $\sin\alpha = \sin\beta$

Prove:  $\alpha - \beta = 2\pi n$  or  $\alpha + \beta = \pi + 2\pi n$ ;  $n \in \mathbb{Z}$

Proof: taken from the condition that  $\sin\alpha - \sin\beta = 0$ , in other words

$2\sin(\alpha - \beta)/2 \cos(\alpha + \beta)/2 = 0$   $\pi n$  or  $(\alpha + \beta)/2 = \pi/2$ , i.e.  $\alpha - \beta = 2\pi n$  or  $\alpha + \beta = \pi + 2\pi n$

Sufficiency proof:

Given:  $\alpha - \beta = 2\pi n$  or  $\alpha + \beta = \pi + 2\pi n$ ;  $n \in \mathbb{Z}$

To prove:  $\sin\alpha = \sin\beta$

Proof: taken from the condition that  $\alpha = \beta + 2\pi n$ , then

$\sin\alpha = \sin(\beta + 2\pi n)$ ,

$\sin\alpha = \sin\beta$  or

$\beta = \pi(2n + 1)$ , then

$\sin\beta = \sin(\pi(2n + 1) - \alpha)$  or

$\sin\beta = \sin[(2\pi n + (\pi - \alpha))] \Rightarrow \sin\beta = \sin(\pi - \alpha)$  in other words

$\sin\beta = \sin\alpha$ . Theorem proven

The students understand  $\cos\alpha = \cos\beta$   $\alpha - \beta = 2\pi n$   $\alpha + \beta = 2\pi n$

Unproven by the same rule (proof must be provided by students). It is given in Theorems 2 and Theorems 3. I mean ( $\sin\alpha = \sin\beta$  @  $\alpha - \beta = 2\pi n$  @  $\alpha + \beta = \pi + 2\pi n$ )

and [ $\operatorname{tg}\beta = \operatorname{tg}\alpha$  @  $\alpha - \beta = \pi n$ ]

After that, with the help of the proved theorems, equations with the condition of equality of trigonometric functions or conditions of equality brought to this point are solved. In our opinion, the simplest trigonometric equations should be solved here. For example

Along with equations of the form  $\sin 3x = \sin x$ ,  $\cos 4x = \cos 2x$ ,  $\operatorname{tg} 5x = \operatorname{tg} x$   
 $\sin 5x = -\sin x$ ;  $\cos 3x = \sin x$

$\operatorname{tg} 3x \cdot \operatorname{tg}(5x + \pi/3) = 1$  solve in equations, and then again propose a

research system.

After explaining all types of equations, the presence of mixed types of equations should also be explained, and at the end it should be drawn up as an independent work covering all types, and when the text of the test is prepared, it is necessary to reflect more types of equations as fully as possible.

### **RESEARCH METHODS / МЕТОДИ ДОСЛІДЖЕННЯ**

To solve the tasks, the following research methods were used:

- analysis of scientific and methodological, psychological, psychological and pedagogical literature;
- the program of the high school mathematics course, analysis of curricula by class;
- analysis of textbooks and teaching aids for teaching mathematics at school;
- analysis of pedagogical and methodological research directly related to problem-based learning;
- study of teachers of higher mathematics and school experience;
- conducting an oral and written survey on the problem under study in grades V-XI;
- conducting a pedagogical experiment in urban, township and rural schools of the republic and achieving a generalization of its results and application in public schools.

To check the learning outcomes of students, modern forms of assessment were used: diagnostic (initial), formative (current) and summative (final).

### **RESEARCH RESULTS / РЕЗУЛЬТАТИ ДОСЛІДЖЕННЯ**

Based on our research and study of pedagogical experience, we can draw the following conclusions:

1. The new Law on Education of the Independent Azerbaijan Republic and the gradual transfer of subjects to the new education system, along with other subjects, was an important stage in the development of secondary mathematical education and has broad prospects.

The goals of mathematical education in the mathematics program for grades V-XI:

- theoretical goal – to give students the necessary mathematical knowledge in life;
- application of mathematical knowledge, mathematical methods in life – in practice: various measurements, calculations, problem solving;

- is defined as the education of spiritual and moral qualities in students by solving specially selected issues and thereby ensuring their comprehensive development. These goals are aimed at connecting mathematics education with life.

2. In the process of solving the problem, the student encounters a problem. To find a solution to this problem, the student:

- tries to determine the place of application of known knowledge;
- tries to discover new knowledge, referring to known knowledge;
- to put forward a judgment, analyze, etc. trying to apply mental operations.

At the same time, the student develops logical, mathematical and dynamic thinking. It is in the process of solving problems that productive teaching methods are used.

3. The connection of mathematical education with life through problem solving – in addition to expanding the general worldview of students, allows you to implement interdisciplinary connections. Because the mathematics course of V–XI grades, being an integrative course, requires the use of developing teaching methods. Since it is impossible to deepen the theoretical level of the course, preference is given to classes of the theoretical-practical, practical and measuring-calculation type.

4. When solving the problem of the connection between teaching mathematics and life through solving problems in grades V–XI, three factors must be taken into account:

1) practice → theory → practice;

2) mathematics as a science is used in solving new practical (as well as theoretical) problems;

3) preparation of a system of tasks taken from life, forming a specific plot, corresponding to the program topics, serving the mathematical and general development of students.

5. The connection of mathematical education with life through solving problems in grades V–XI depends on three factors:

- scientific and methodological preparation and initiative of the teacher;
- in the development of existing textbooks in mathematics;
- Availability of additional educational (didactic) materials for teachers and students in the mathematics course V–XI grades.

There are objective and subjective reasons why mathematics textbooks in grades V–XI have little access to the presented problem. However, the task of activating the informativity of learning directly depends on the business acumen and pedagogical skills of the teacher.

6. In the course of mathematics in grades V–XI, students get acquainted with arithmetic operations on the set of rational numbers, the material of the preparatory course of geometry in the direction of «practice-theory» and learn simple geometric constructions. This knowledge allows teaching methods for solving various types of problems (including geometric ones).

Although this is not the first attempt to connect the teaching of mathematics with life through problem solving, a methodological system for solving this problem has been developed, taking into account the peculiarities of the mathematics course in grades V–XI, as well as on local materials, not only problems have been identified, but in general a methodological system has been developed, the content is indicated work, methods of implementation, means, based on the assessment of learning outcomes, theoretical and practical significance and methodological research were confirmed.

7. For each of the subjects related to the school mathematics course (including the interactive course), various types of classes are used to assimilate theoretical material, consolidate it and implement its application in practice. This is the minimum position for teaching this subject. However, there are issues that allow the implementation of several didactic functions. This includes standard and non-standard questions that provide the student with additional (non-math) information. Since the method of solving a problem is related to the content of the problem, it remains in the memory of students.

## **CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH / ВИСНОВКИ ТА ПЕРСПЕКТИВИ ПОДАЛЬШИХ ДОСЛІДЖЕНЬ**

To improve the quality of mathematical education in grades V–XI, it is not enough to improve the content and form of the methodological system of teaching the subject; it is necessary to take into account the requirements of the new educational system (curriculum), the individual characteristics and capabilities of students, and use interactive teaching methods. Because a math problem is a means of attacking the student's brain.

**Prospects for further research in this direction.** «Statistics and Probability» – being a new meaningful direction in the school mathematics course, her teaching in grades V–XI is based mainly on pressing problems. Statistical elements were used in school practice. However, in the new education system, both concepts should be taught in an interrelated way and with the help of appropriate questions.

It is necessary to improve the technology for solving problems in statistics and probability in school mathematics. Since the possibilities for solving

different types of problems in the school mathematics course are available in grades V–XI, then in order to implement its didactic functions, it is necessary to allocate enough space for solving problems in the classroom and after school hours cool exercises.

The educational performance of students in the experimental classes was higher than in other subjects.

#### REFERENCES (TRANSLATED AND TRANSLITERATED) / СПИСОК ВИКОРИСТАНИХ ДЖЕРЕЛ

- [1] M. C. Abdulkərimov, «Orta məktəbin riyaziyyat kursunda ekstremum və tətbiq məsələləri təlimin həyatla əlaqələndirilməsi vasitəsi kimi (V–XI siniflər)», namiz. dis. Bakı: ADPU, 2011.
- [2] M. M. Aşurov, «Səkilillik məktəbin cəbr kursunda tətbiq məsələləri təlimin keyfiyyətini artıran vasitə kimi», namiz. dis. avtoref. Bakı: APİ, 1990.
- [3] L. İ. Qocayeva, «Məsələ həlli vasitəsilə riyaziyyat təliminin həyatla əlaqələndirilməsi (III–IV siniflər)», namiz. dis. avtoref. Bakı: ADPU, 2001.
- [4] B. Q. Əliyev, *Ali pedaqoji məktəblərdə gələcək ibtidai sinif müəllimlərinin riyaziyyatdan elmi-metodiki hazırlığının təmin edilməsində məsələ bir vasitə kimi*. Bakı, Azərbaycan: ADPU, 2001.
- [5] S. S. Həmidov, *I–IV siniflərdə riyaziyyatın tədrisi metodikası*. Bakı, Azərbaycan: ADNA, 2008.
- [6] S. S. Həmidov, N. M. Hacıyev, *İbtidai siniflərdə məsələ həlli təliminin nəzəri metodiki problemləri*. Bakı, Azərbaycan: ADNA, 2008.
- [7] S. S. Həmidov, *Məktəbin ibtidai siniflərində məsələ həllinin təlimi metodikası*. Bakı, Azərbaycan: Natural, 2003.
- [8] S. S. Həmidov, *Pedagoji fakültədə riyaziyyatın tədrisi metodikası*. Bakı, Azərbaycan: Bakı APİ, 1990.
- [9] S. R. Hüseynova, *Riyaziyyatdan ümumiləşdirici və inkişafetdirici məsələlər vasitəsilə nəzəri biliklərin verilməsi və möhkəmləndirilməsi (VI–XI siniflər)*. Bakı, Azərbaycan: ADPU, 2003.
- [10] A. N. İsmayılov, *Orta ümumtəhsil məktəbi şagirdlərinin riyaziyyatdan və fizikadan nəzəri biliklərin kənd təsərrüfatına tətbiqi üzrə işin sistemi*. Bakı, Azərbaycan: APİ. 1996.
- [11] N. Qəhrəmanova, C. Əsgərova, *Riyaziyyat – 3*. Bakı, Azərbaycan: Altun kitab, 2008.
- [12] N. Qəhrəmanova, C. Əsgərova, *Riyaziyyat – 4*. Bakı, Azərbaycan: Altun kitab, 2009.
- [13] Ə. A. Quliyev, *Planimetriyanın məsələ vasitəsilə təkrarı*. Bakı, Azərbaycan:


Nurlan, 2008.

- [14] Ə. A. Quliyev, *Riyaziyyatdan tədrisində ümumiləşdirmə*. Bakı, Azərbaycan: Elm, 2009.
- [15] M. C. Mərdənov, və b. *Riyaziyyat – 6*. Bakı, Azərbaycan: Çarşıoğlu, 2008.
- [16] M. V. Namazov, *Riyaziyyatdan çalışmalar. VI sinif üçün vəsait*. Bakı, Azərbaycan: Dərələyəz-M, 2011.
- [17] İ. M. Rüstəmov, «Həndəsədən məsələ həlli vasitəsilə şagirdlərin nəzri biliklərinin inkişaf etdirilməsi (VII–IX siniflər)», namizədlik dis. Avtoref. Bakı: APİ, 1997.

## ОПТИМАЛЬНІ ФОРМИ НАВЧАННЯ ТЕМИ «ТРИГОНОМЕТРИЧНІ РІВНЯННЯ» У ШКІЛЬНОМУ КУРСІ МАТЕМАТИКИ

**Джафарлі Есмiра Вахiд,**

Доктор філософії з педагогіки,  
доцент Гянджинського державного університету.  
Баку, Азербайджан.

 <https://orcid.org/0000-0003-3619-4356>  
[gulizade.gunel@mail.ru](mailto:gulizade.gunel@mail.ru)

**Мамедова Джабаруд Музаффар,**

старший викладач Гянджинського державного університету.  
Баку, Азербайджан.

**Мустафаева Парвин Маил,**

викладач Гянджинського державного університету.  
Баку, Азербайджан.

**Анотація.** Обрану для дослідження статтю присвячено оптимальній методиці навчання тригонометричних рівнянь у курсі математики середньої школи на основі навчальної програми. У статті також підтверджується, що вивчений матеріал є логічним завершенням курсу тригонометрії, підсумковим результатом пройдених теоретичних знань. Тема тригонометричних рівнянь і нерівностей надає чудові приклади та проблеми інтеграції між алгеброю, геометрією та іншими дисциплінами. Тригонометрія за своїми характеристиками тісно пов'язана як з геометрією, так і з алгеброю, і цей зв'язок відображено в змісті шкільного курсу математики. Таким чином, тригонометрія використовується у геометрії в метричних співвідношеннях у прямокутних трикутниках, у визначенні синуса, косинуса, тангенса і котангенса гострого кута, у теоремі синусів і косинусів, у формулах співвідношення між сторонами і радіусами,

коли кола проведені всередині і зовні правильні багатокутники, у формулах площ багатьох геометричних фігур тощо, в алгебрі і в курсі наведені в розділах основні формули тригонометрії, тригонометричні рівняння і нерівності тощо. Наша головна мета дослідження – пояснити місце і значення тригонометричних рівнянь і нерівностей у шкільному курсі математики, показати роль тригонометричних рівнянь, визначити методичні особливості їх розв'язання. Водночас у процесі навчання тригонометричним рівнянням і нерівностям розглядається і розв'язування алгебраїчних рівнянь, що входять до системи рівнянь, а це вимагає як від учителя, так і від учня викладати та вивчати курс математики середньої школи у формі, повний, цілісний спосіб.

**Ключові слова:** курс математики; тригонометричні рівняння; освітній процес; студенти; викладач; алгебра; геометрія; школа.

#### **REFERENCES (TRANSLATED AND TRANSLITERATED)**

- [1] M. C. Abdulkerimov, "Extremum and application issues in the high school mathematics course as a means of connecting learning with life (grades V–XI)", cand. dis. Baku: ADPU, 2011.
- [2] M. M. Ashurov, "Application problems in the algebra course of the eight-year school as a means of increasing the quality of education", cand. dis. autoref. Baku: API, 1990.
- [3] L. I. Gocayeva, "Connecting mathematics teaching with life through problem solving (III–IV grades)", cand. dis. autoref. Baku: ADPU, 2001.
- [4] B. Q. Aliyev, the issue as a tool in ensuring the scientific-methodical training of future elementary school teachers in mathematics. Baku, Azerbaijan: ADPU, 2001.
- [5] S. S. Hamidov, Mathematics teaching methodology in grades I–IV. Baku, Azerbaijan: ADNA, 2008.
- [6] S. S. Hamidov, N. M. Hajiyev, Theoretical and methodological problems of problem solving training in primary classes. Baku, Azerbaijan: ADNA, 2008.
- [7] S. S. Hamidov, Methodology of teaching problem solving in primary classes of the school. Baku, Azerbaijan: Natural, 2003.
- [8] S. S. Hamidov, Methodology of teaching mathematics at the Faculty of Pedagogy. Baku, Azerbaijan: Baku API, 1990.
- [9] S. R. Huseynova, Providing and strengthening theoretical knowledge through summarizing and developing problems in mathematics (grades



- VI–XI). Baku, Azerbaijan: ADPU, 2003.
- [10] A. N. Ismayilov, The system of work on the application of theoretical knowledge of mathematics and physics to agriculture of secondary school students. Baku, Azerbaijan: API, 1996.
- [11] N. Kahramanova, C. Asgarova, Mathematics – 3. Baku, Azerbaijan: Altun kitab, 2008.
- [12] N. Kahramanova, C. Asgarova, Mathematics – 4. Baku, Azerbaijan: Altun kitab, 2009.
- [13] A. A. Guliyev, Repetition of planimetry through the problem. Baku, Azerbaijan: Nurlan, 2008.
- [14] A. A. Guliyev, Generalization in teaching mathematics. Baku, Azerbaijan: Elm, 2009.
- [15] M. C. Mardanov, and others. Mathematics – 6. Baku, Azerbaijan: Çashioğlu, 2008.
- [16] M. V. Namazov, Studies in mathematics. Materials for class VI. Baku, Azerbaijan: Deraleyaz-M, 2011.
- [17] I. M. Rustamov, "Development of theoretical knowledge of students through problem solving in geometry (VII–IX classes)", candidate diss. Autoref. Baku: API, 1997.

*Стаття надійшла до редакції  
28 грудня 2022 року*